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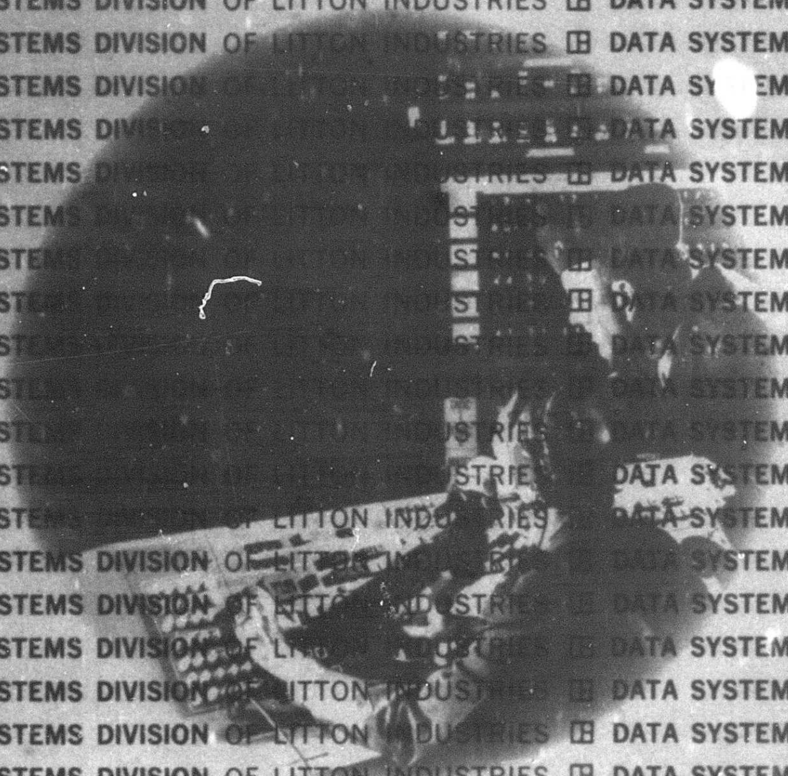

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THEORETICAL AND EMPIRICAL RESULTS ON THE CHARACTERIZATION OF UNDERSEA ACOUSTIC CHANNELS*

by

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ABSTRACT

An undersea acoustic link between two points in space is represented as a random time-varying linear filter. It can therefore be characterized by its impulse response, or alternately by various Fourier transforms, known as the transfer function, spreading function, and bi-frequency function. All four of these descriptors are random functions of two variables. A generalization of the uncorrelated scattering model is proferred and it is shown how it can fit known ocean characteristics.

Results are presented of some measurements made of the time and frequency spreading imposed on audio frequency transmissions over open ocean underwater paths. Most of the results deal with frequency spreading; there are a few fine grain time domain measurements. (The major features of the time domain behavior are already part of the literature.) Results are given for several center frequencies ranging from the low hundreds to the low thousands of cycles per second. Paths between both fixed and mobile end points have been measured. The path lengths used have ranged from one-half to several hundreds of miles.

The results indicate that many of the simplifying assumptions usually made in discussions of time varying channels are not satisfied by these paths, and these points where the assumptions must be generalized are briefly discussed.

1. Theoretical Results on the Characterization of the Undersea Acoustic Channel

1.1 A Linear Time-Varying Stochastic Model

The simplest, reasonably adequate, model of an undersea acoustic channel is that of a linear time-varying stochastic filter, where the filter, of course, depends on the location of the transmitter and receiver and the intervening medium, surface, and bottom. The linearity assumption allows for the use of superposition in determining the response of the acoustic channel to several simultaneous inputs, knowing only the response to one input, namely the impulse response, while the time-varying stochastic features allow a multipath structure to change with time in a random unpredictable way.

There are several equivalent ways of characterizing a random linear time-varying filter. The first, mentioned above, is the response at time t to a unit impulse applied τ ago, and is called the impulse response $h(\tau, t)$. For a given location and geometry, and given values of τ and t, this quantity must be considered a random variable; i.e., $h(\tau, t)$ is a member of an ensemble. For fixed t, the extent of $h(\tau, t)$ on the τ scale measures the amount of time delay spread a signal undergoes in passage through the filter, while for fixed τ, the behavior of $h(\tau, t)$ with t indicates how rapidly the filter characteristics are changing with time, and hence how much frequency shift spread occurs. For a general input x(t), the output y(t) of the filter is given by*

$$y(t) = \int d\tau \; h(\tau, t) \; x(t-\tau). \tag{1}$$

As an example of the above, fluctuating multipath might be well characterized by $h(\tau, t) = \sum_k g_k(t)\, \delta(\tau - \tau_k)$, where $g_k(t)$ and τ_k are the time-varying gain and (stable) delay of the k-th path.

A more convenient description than the impulse response for undersea purposes is given by the Fourier transform of $h(\tau, t)$ on t, and is called the

*Integrals without limits are over the range of non-zero integrand.

spreading function:

$$a(\tau, \eta) = \int dt \exp(-i2\pi\eta t)h(\tau, t). \tag{2}$$

The best way of illustrating the utility of the spreading function is to note how it transforms an input waveform x(t). By substituting the inverse transform relation of eq. (2) into eq. (1), we find we can write

$$y(t) = \iint d\tau\, d\eta \left[x(t-\tau)\exp(i2\pi\eta t)\right] a(\tau, \eta). \tag{3}$$

Equation (3) says that the total output waveform is obtained by summing time-delayed and frequency-shifted versions of the input waveform (in brackets), weighted by the spreading function at each value of delay and shift. Thus $a(\tau, \eta)$, for specific values of τ and η, measures how much simultaneous time delay τ and frequency shift η any input signal will be subject to. It must, of course, be considered random, just as the impulse response is.

Alternatively, if we let the spectra (Fourier transforms) of x(t) and y(t) be donoted by $S_x(f)$ and $S_y(f)$, we find, by transforming eq. (3), that

$$S_y(f) = \iint d\tau d\eta \left[S_x(f-\eta)\exp\left\{-i2\pi(f-\eta)\tau\right\}\right] a(\tau, \eta). \tag{4}$$

Since the bracketed quantity is the spectrum of a time-delayed and frequency-shifted input signal, eq. (4) says that the output spectrum is obtained by weighting this quantity by the spreading function and then summing. Thus once again, $a(\tau, \eta)$ is interpreted as the amount of signal undergoing delay τ and shift η.

The total extent of $a(\tau, \eta)$ on the τ, η plane measures the amount of spreading any signal will incur on passage through the filter. The extent on τ, called the time delay spread, will be denoted by the symbol L, and the extent on η, called the frequency shift spread, will be denoted by B. (Actually since $a(\tau, \eta)$ is random, i.e., is a member of an ensemble, L and B must be determined as ensemble average parameters; this will be elaborated later.) The product BL (dimensionless) measures the area of spread in the delay-shift (τ, η) plane, and is one of the most important parameters of a time varying random filter. Note that $a(\tau, \eta)$ may consist of discrete "lumpy" contributions at widely separated points of the τ, η plane; that is, L and B represent the distance between extremities of a (possibly) multimodal function.

Before continuing with our discussion of the spreading function, we list two other characterizations of a linear time-varying random filter that are occasionally of use. The first is called the instantaneous transfer function and is given by

$$H(f, t) = \int d\tau \exp(-i2\pi f \tau)\, h(\tau, t). \qquad (5)$$

Its interpretation and utility comes about in that $H(f_o, t)$ gives the ratio of output to input at time t when the input is $x(t) = \exp(i2\pi f_o t)$ for all time. (Actually the input need have been $\exp(i2\pi f_o t)$ only for approximately L seconds, because the filter memory is no longer than this). Thus $H(f, t)$ at fixed t gives the instantaneous amplitude gain and phase shift of each component of the input spectrum. The output waveform of such a filter can therefore be expressed as

$$y(t) = \int df \exp(i2\pi ft)\, S_x(f)\, H(f, t). \qquad (6)$$

The last quantity of interest is called the bi-frequency function

$$\begin{aligned} A(f, \eta) &= \int dt \exp(-i2\pi \eta t)\, H(f, t) \\ &= \int d\tau \exp(-i2\pi f \tau)\, a(\tau, \eta) \\ &= \iint d\tau\, dt \exp\left[-i2\pi (f\tau + \eta t)\right] h(\tau, t). \end{aligned} \qquad (7)$$

Its interpretation is made easy by consideration of the output spectrum for a general input spectrum:

$$S_y(f) = \int d\eta\, S_x(f-\eta)\, A(f-\eta, \eta). \qquad (8)$$

This equation says that the output spectrum at frequency f is determined by weighting input spectrum components at frequency $f-\eta$, (i.e., a shift of η), by the value of the bi-frequency function at that frequency and with that shift, and summing. Thus $A(f, \eta)$ measures the amount of input signal spectrum at frequency f undergoing shift η.

It is seen that all four (random) characterizations described above can be useful for discussion and description of linear random time-varying filters in one context or another. In Figure 1, slices of the four functions in the various domains have been depicted, and two new symbols have been added. F is the bandwidth of the filter (that region of frequencies passed by the filter; see Fig. 1(b)), and D is the duration or "lifetime" of the filter (i.e., how long the filter exists in time, and can be considered infinite; (see Fig. 1(a)). The distances marked off on the various axes indicate how rapidly a significant change in value of each function <u>can</u> occur, (not necessarily how rapidly it actually does occur.*) Figure 1 must be considered as one member of an ensemble with random properties. Thus, slices of other members of the ensemble would have different values, but would extend over roughly the same regions of the planes and change no faster than in the distances indicated.

All the above has been couched in terms of the real input waveform x(t). However for narrow-band inputs**, it is convenient to represent x(t) as amplitude and phase modulation on a "carrier" according to complex notation:

$$x(t) = \mathrm{Re}\left\{\underline{x}(t) \exp(i2\pi f_o t)\right\} , \qquad (9)$$

where f_o is the input center frequency, Re denotes "real part of", and $\underline{x}(t)$, the complex envelope, has a spectrum confined to frequencies around zero. Since we can do the same for a random time-varying filter (at least in the range of frequencies covering the input signal band), we can express

$$h(\tau, t) = \mathrm{Re}\left\{\underline{h}(\tau, t) \exp(i2\pi f_o \tau)\right\} , \qquad (10)$$

where

$$\underline{H}(f, t) = \int d\tau \exp(-i2\pi f \tau)\, \underline{h}(\tau, t) \qquad (11)$$

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*When the bandwidth F of the filter is centered about some high carrier frequency, the variation with τ should be interpreted as the complex envelope of the waveform.

**Actually, the narrow-band requirement is not necessary. Single-sided spectra and corresponding complex envelope signals can be defined and used for wide-band signals also.

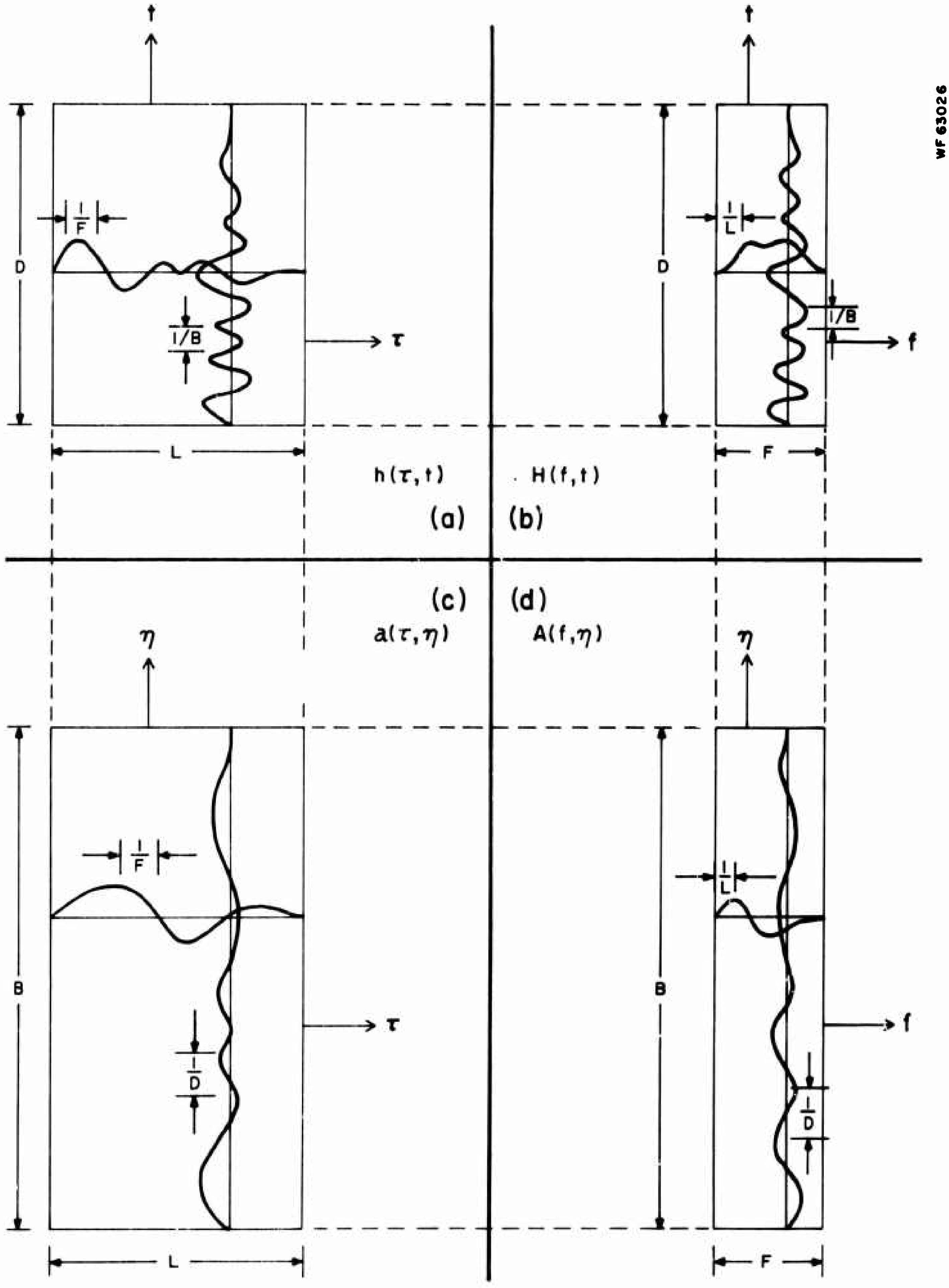

Figure 1. FILTER FUNCTIONS

is confined around zero frequency. It can then be shown that except for irrelevant scale factors, the complex envelope of the filter output is given by

$$\underline{y}(t) = \int d\tau\, \underline{h}(\tau, t)\, \underline{x}(t-\tau)$$

$$= \iint d\tau\, d\eta\, \underline{x}(t-\tau)\, \exp(i2\pi\eta t)\, \underline{a}(\tau, \eta). \qquad (12)$$

Thus, just as in eqs. (1) and (3), the complex low frequency input waveform and filter low-frequency descriptors suffice to determine the output.

Upon reception of the waveform distorted by an undersea acoustic channel, linear processing, in the form, perhaps, of noise rejection filtering or matched filtering, is often employed. Then we can represent the (assumed time-invariant) receiver filtering operation by its impulse response according to

$$h_r(\tau) = \mathrm{Re}\left\{ \underline{s}^*(-\tau)\, \exp\left[i2\pi(f_o + f_s)\tau \right] \right\}. \qquad (13)$$

Here $\underline{s}$ is a low frequency time function, and f_s is a local frequency shift injected to try to compensate for any unknown doppler shift caused by the acoustic channel. In this formulation, we are lumping medium and target together and calling them a channel. (We have anticipated matched filtering or some approximation to it by substituting the negative argument and conjugate on $\underline{s}$; however, by choice of $\underline{s}$ with arbitrary delay, actually any filter is allowed.) It may be then shown that the complex envelope of the output of the filter of eq. (13) for the input of eq. (12) is given by

$$\underline{z}(t) = \iint d\tau\, d\eta\, \exp(i2\pi\eta t)\, \underline{a}(\tau, \eta)\, \chi^*(t-\tau, f_s-\eta), \qquad (14)$$

where $\chi(\tau, \eta)$ is the cross-ambiguity function of transmitted signal and receiving filter:

$$\chi(\tau, \eta) = \int du\, \exp(i2\pi\eta u)\, \underline{s}(u)\, \underline{x}^*(u + \tau). \qquad (15)$$

Thus the output waveform is obtained as a double convolution of the spreading function with the cross-ambiguity function. Eq. (14) enables us to find exactly how the spreading function degrades the response of the receiver from its ideal output, for one particular transmission. Since $\underline{a}(\tau, \eta)$ is random, so too is $\underline{z}(t)$.

The generality of the above approach is great enough to include both one-way and two-way paths, large moving fluctuating targets, reverberation, and multiplicative noise. For example, a moving target with several strong highlights in the presence of reverberation could be represented as

$$\underline{a}(\tau, \eta) = \delta(\eta - \eta_d) \sum_k A_k \delta\left(\tau - \tau_{d_k}\right) + \sum_j a_j \delta(\tau - \tau_j)\, \delta(\eta - \eta_j), \tag{16}$$

where η_d is the target doppler shift, $\{A_k\}$ and $\left\{\tau_{d_k}\right\}$ are the relative strengths and delays of the individual highlights, and $\{a_j\}$, $\{\tau_j\}$, and $\{\eta_j\}$ are the random strengths, delays, and shifts of a multitude of point reflectors. Additive sea noise and platform noise must be added to the received signal of Eq. (12) to find their effects on the output of the receiver.

1.2 Average Descriptors of the Linear Model

The significant amount of fine detail about one member of an ensemble depicted in Figure 1 is sometimes not of interest, not available, or impossible to measure. Accordingly some simple average descriptors of system performance and their effect on input waveforms is often desirable. One of the most reasonable requirements about statistics of the filter output would be to know its autocorrelation function. Since the output process can be generally non-stationary, even though the input may be stationary, we are asking, in general (for a deterministic input), for the quantity

$$\overline{\underline{y}(t_1)\, \underline{y}^*(t_2)} = \iint d\tau_1\, d\tau_2\, \overline{\underline{h}(\tau_1, t_1)\, \underline{h}^*(\tau_2, t_2)}\, \underline{x}(t_1 - \tau_1)\, \underline{x}^*(t_2 - \tau_2), \tag{17}$$

where the overbars signify an ensemble average, and we have employed Eq. (1). Thus the autocorrelation of the impulse response of the filter is sufficient knowledge to evaluate the output correlation. Since the spreading function, transfer function, and bi-frequency function are all Fourier transforms of each other, knowledge of the correlation function of any one of them is tantamount to knowledge of all the others; thus measurement or estimation can be effected on any desired characterization, depending on convenience and measurement technique.

The simplest possible assumption about the spreading effect of the undersea acoustic channel is to assume that the value of the spreading function at each time delay and frequency shift is uncorrelated with the value at any other delay and/or shift. That is, one assumes that*

$$\overline{\underline{a}(\tau_1, \eta_1)\, \underline{a}^*(\tau_2, \eta_2)} = \sigma(\tau_1, \eta_1)\, \delta(\tau_1 - \tau_2)\, \delta(\eta_1 - \eta_2). \tag{18}$$

Here $\sigma(\tau, \eta)$ is called the scattering function and is a measure of the average amount of signal power undergoing delay τ and shift η. How useful an assumption Eq. (18) is for the undersea acoustic channel is yet to be determined. However some important ramifications of this uncorrelated spreading assumption should be pointed out. First, it can be shown that the ensemble average power transmission of the filter is independent of frequency and time. That is, Eq. (18) leads to the requirement that

$$\overline{|\underline{H}(f, t)|^2} = \iint d\tau\, d\eta\, \sigma(\tau, \eta). \tag{19}$$

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*Price, R. and Green, P.E., Jr., "Signal Processing in Radar Astronomy - Communication via Fluctuating Multipath Media," Lincoln Laboratory, T.R. No. 234, Mass. Inst. of Tech; Oct. 6, 1960.

(Recall that $f = 0$ here corresponds to the center frequency of the transmitted signal.) For sufficiently narrow-band input signals and short observation times, this can be an adequate description for many channels. However, for other situations, either wide-band or long duration signals, it should be generalized, especially for the undersea acoustic channel. (Incidentally, Eq. (19) says that the volume under the scattering function equals the average power transmission of the filter.)

A more general result that must follow from the uncorrelated spreading assumption of Eq. (18) is that

$$\overline{\underline{H}(f_1, t_1)\, \underline{H}^*(f_2, t_2)} = \Lambda(f_1 - f_2,\ t_1 - t_2), \tag{20}$$

where $\Lambda(f, t)$ is the double Fourier transform of $\sigma(\tau, \eta)$:

$$\Lambda(f, t) = \iint d\tau\, d\eta\, \exp\left[-i2\pi\,(f\tau - \eta t)\right] \sigma(\tau, \eta). \tag{21}$$

If, as above, it is assumed that $\sigma(\tau, \eta)$ has extent L by B in the τ, η plane, then $\Lambda(f, t)$ can change significantly in distances no smaller than $1/L$ by $1/B$ respectively in the f, t plane. Equation (20) then says that the ensemble average of the product of the transfer function at different frequencies and times depends only on the difference of the two frequencies and times, and can be significantly decorrelated for frequency separations as small as $1/L$ and time separations as small as $1/B$. (However, $\Lambda(f, t)$ can have extensive skirts of low level, indicating small correlation.)

The uncorrelated spreading assumption also leads to a convenient measure of the effect of the channel on the output of a matched filter, or more generally, any linear processor. For example, the ensemble average value of $|\underline{z}(t)|^2$ of Eq. (14) gives the average value of the square of the envelope of the output of the receiving filter as a function of time. It is, employing Eq. (18),

$$\overline{|\underline{z}(t)|^2} = \iint d\tau\, d\eta\, \sigma(\tau, \eta)\, |\chi(t - \tau, f_s - \eta)|^2. \qquad (22)$$

That is, the average power output is the double convolution of the scattering function with the magnitude-squared cross-ambiguity function. Thus a very concentrated scattering function (small L, B) would effectively sample the peak value of the cross-ambiguity function with no degradation of performance, while a broad scattering function would smooth out the cross-ambiguity function, yielding smaller average outputs (for a fixed volume under the scattering function, i.e., a fixed average power gain of the filter.) Thus the loss of correlation* due to spreading can be computed from this equation.

The notion of a concentrated scattering function can be made quantitative as follows: for a signal of duration T and bandwidth W, the widths of the cross-ambiguity function in the τ and η directions are $1/W$ and $1/T$, respectively, if matched filtering is employed. Therefore, <u>if</u> $L < 1/W$, and $B < 1/T$, we are guaranteed very little loss due to spreading. Thus a scattering function can be considered concentrated for some signals and not for others, depending on the signal time duration and bandwidth. In any event, since for any signal, $TW > 1$, we must have $BL < 1/TW < 1$ in order to have negligible degration due to spreading. However, $BL < 1$ is not sufficient to guarantee no spreading loss.

As mentioned in connection with Eq. (19), one shortcoming of the uncorrelated spreading assumption is that the average power gain of the filter must be independent of frequency. In an attempt to relax this restriction, the following spreading dependence, which seems reasonable for many random channels, is proferred. Namely, the values of the spreading function at different delays and shifts are partially correlated, but can become uncorrelated for delay and/or shift separations small

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*Weston, D.E., "Correlation Loss in Echo Ranging," JASA, Vol. 37, No. 1, pp. 119-124; Jan. 1965 compared to the total delay spread L and the frequency shift spread B. That is, we assume

$$\overline{\underline{a}(\tau_1, \eta_1)\, \underline{a}^*(\tau_2, \eta_2)} = \sigma\left(\frac{\tau_1 + \tau_2}{2}, \frac{\eta_1 + \eta_2}{2}\right) g(\tau_1 - \tau_2, \eta_1 - \eta_2), \tag{23}$$

where the widths of the peak(s) of $g(\tau, \eta)$ in τ and η, which measure the interdependence of different delays and shifts on one another, can be small compared to L and B. $g(\tau, \eta)$ is called the interaction function. The value of the scattering function $\sigma(\tau, \eta)$ again is a measure of the average signal power suffering delay τ and shift η. The assumption of Eq. (23) is called locally stationary;* it is perhaps most easily interpreted in the (low frequency equivalent) transfer function domain, where Eq. (23) is tantamount to

$$\overline{\underline{H}(f_1, t_1)\, \underline{H}^*(f_2, t_2)} = \Lambda(f_1 - f_2, t_1 - t_2)\, C\left(\frac{f_1 + f_2}{2}, \frac{t_1 + t_2}{2}\right). \tag{24}$$

Here $\Lambda(f, t)$ is again given by Eq. (20), and $C(f, t)$ is the double Fourier transform of the interaction function $g(\tau, \eta)$. If $\sigma(\tau, \eta)$ still has total extent L by B in the τ, η plane, Eq. (24) indicates that filter transmissions at frequencies and times separated by $1/L$ and $1/B$, respectively, can be uncorrelated. For closer spacings, the amount of filter transmission depends on the exact frequency and time according to the function $C(f, t)$. In fact, as a special case of Eq. (24), the ensemble average power transmission is

$$\overline{|\underline{H}(f, t)|^2} = \iint d\tau\, d\eta\, \sigma(\tau, \eta)\, C(f, t). \tag{25}$$

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*Silverman, R.A., "Locally Stationary Random Processes," IRE Trans. on Info. Th., Vol. IT-3, No. 3, pp. 182-187; Sept. 1957.

Thus the extent of $C(f, t)$ on f measures the bandwidth of the filter (F of Figure 1) while the extent of $C(f, t)$ on t measures the time duration (existence) of the filter (D of Figure 1 which is generally infinite. Actually, there are cases, such as a submarine moving in and out of a shadow zone, where D is finite.)

The correlated-spreading assumption allows us to have a filter whose average transmission depends on frequency and on time. For a large number of undersea situations, say for limited observation times of fairly wide-band signals, $C(f, t)$ may be assumed independent of t, but the frequency dependence kept. In this case, we obtain a spreading assumption which amounts to uncorrelated spreading at different shifts but correlated spreading in delay, with peaks in the interaction function $g(\tau, \eta)$ as narrow as $1/F$ in τ.

At this point it is well to summarize the correlated-spreading assumption, and depict the various average functions as was done for the random functions in Figure 1. In Figure 2 is shown the two dimensional slices of the scattering function $\sigma(\tau, \eta)$ and its various Fourier transforms, while in Figure 3, the same treatment is given to the interaction function $g(\tau, \eta)$. It is seen that the total extent of the scattering function is L by B in the τ, η plane. However, there is allowed the possibility that $\sigma(\tau, \eta)$ will change much more rapidly than this; that is, $\sigma(\tau, \eta)$ is not restricted to be unimodal. If the transfer function has partial correlation for frequency separations up to f_c and for time separations up to t_c [See Figure 2(b)], then the scattering function can change significantly in distance $1/f_c$ by $1/t_c$. The possible rates of change of the other various transforms are also indicated.

In Figure 3(b), we have assumed the filter bandwidth to be F and the duration D. Therefore the interaction function $g(\tau, \eta)$ can change significantly in distances no smaller than $1/F$ by $1/D$ as indicated in Fig. 3(c). However, although the interaction function may be sharp about the origin,

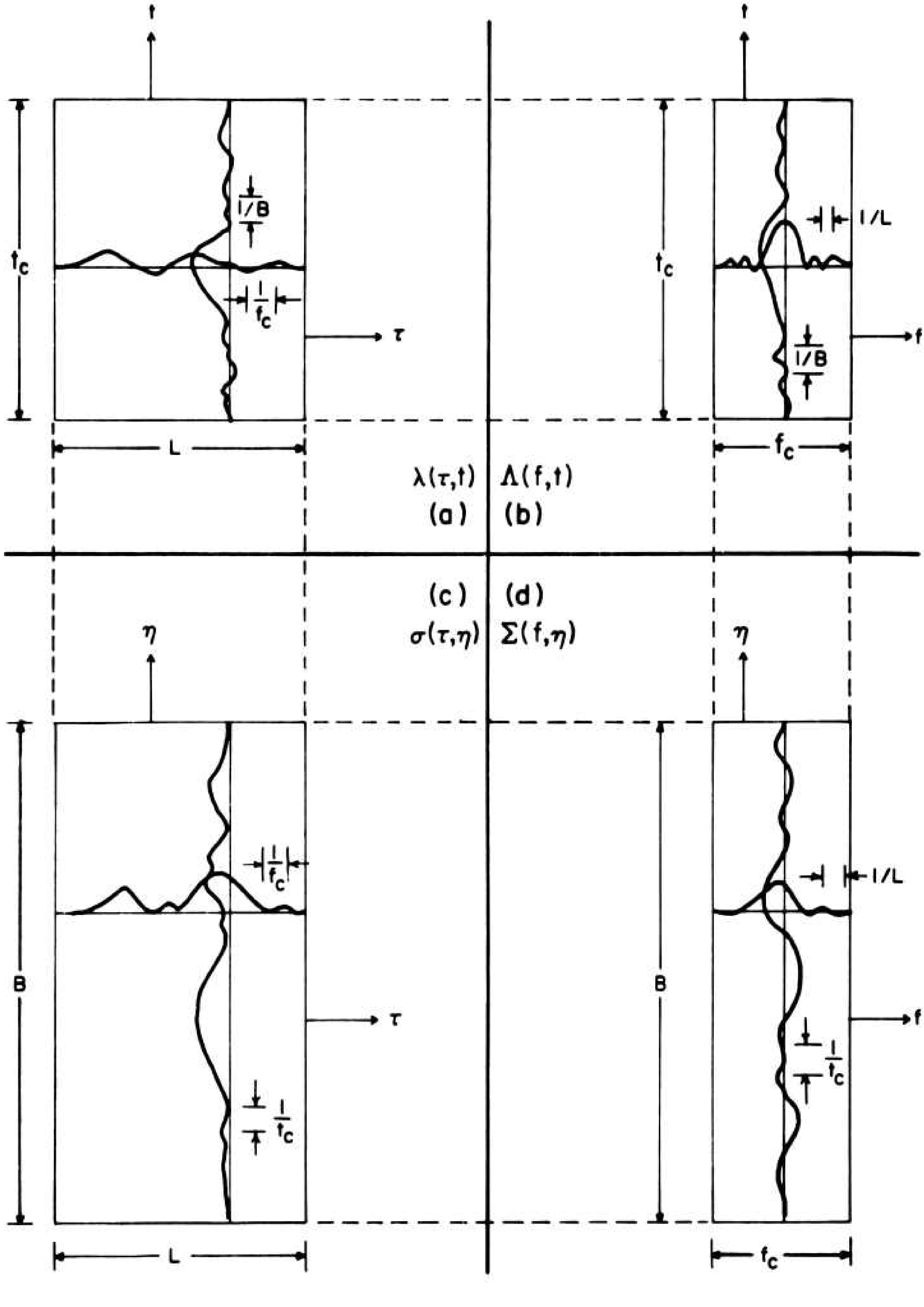

Figure 2.
SCATTERING FUNCTION and TRANSFORMS

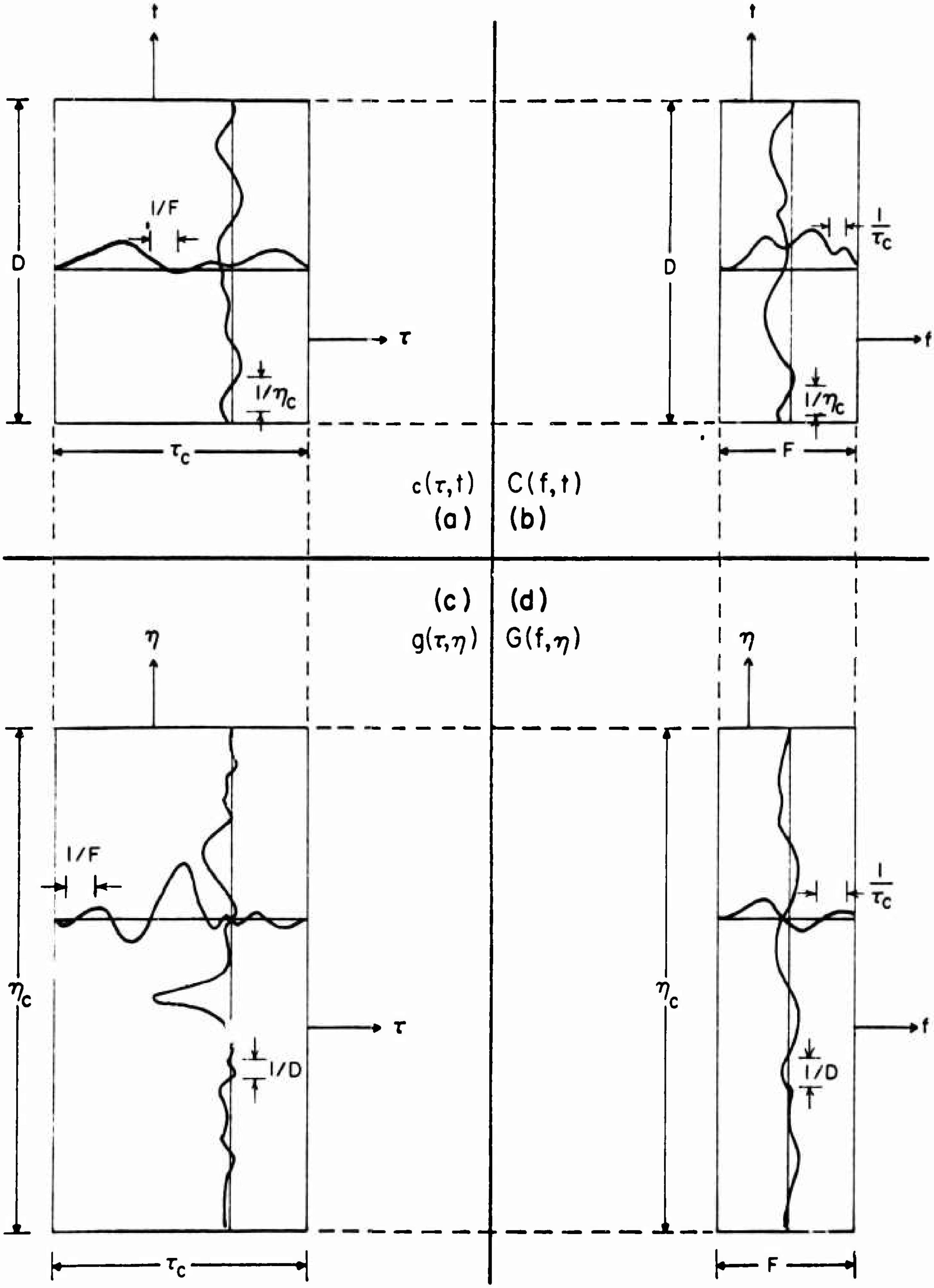

Figure 3.
INTERACTION FUNCTION and TRANSFORMS

it need not be unimodal, but may have partial correlation values over a range τ_c by η_c. This allows the average power gain of the filter Fig. 3(b) to change significantly in distance $1/\tau_c$ by $1/\eta_c$ as indicated. It must be stressed that $g(\tau, \eta)$ need not decay to zero for $|\tau| > 1/F$; rather it can decay to zero no sooner than in $1/F$, and may in fact be near its peak for many times the interval $1/F$. A similar comment holds with respect to the decay of $g(\tau, \eta)$ with η.

The effect of a correlated-spreading channel on the output of a linear processor is obtained by substituting Eq. (23) into the mean value of the squared magnitude of Eq. (14) obtaining

$$\overline{|z(t)|^2} = \iint d\tau\, d\eta\, \sigma(\tau, \eta)\, A(t - \tau, f_s - \eta; t), \tag{26}$$

where

$$A(\tau, \eta; t) = \iint du\, dv \exp(i2\pi vt)\, g(u, v)\, \chi^*(\tau - \frac{u}{2}, \eta - \frac{v}{2}) \cdot (\tau + \frac{u}{2}, \eta + \frac{v}{2}). \tag{27}$$

Thus the average envelope-squared value of the linear processor output is given by the double convolution of the spreading function with a local average of the cross-ambiguity function. Since the bandwidth of the medium will be greater than that of the signal being transmitted through it, $F > W$, the width of the interaction function $g(\tau, \eta)$ can be narrower than that of the χ-function in the τ direction: $1/F < 1/W$. Similarly, the width of $g(t, \eta)$ can be narrower than the χ-function in the η direction. These effects alone would cause only slight widening of the averaged ambiguity function A. However, the extent and volume of the low lying skirts of $g(\tau, \eta)$ (out to τ_c, η_c) cause a further smoothing which can be very important.

As an application of Eqs. (26) and (27) to a correlated-scattering channel, let us consider the case where the average transmission of the filter varies across the bandwidth, but is constant with time. That is, we assume

$$C(f, t) = 1 + \alpha \cos(\pi f \tau_c + \theta) \tag{28}$$

over the range of frequencies occupied by the input signal. [The frequency interval $1/\tau_c$ is the distance between a peak and valley of C(f, t); see Figure 3(b)]. The value of α measures the amount of variation of the transmission function, while θ places the peak of the frequency variations at an arbitrary frequency with respect to the input signal bandwidth. (Recall that f = 0 corresponds to the center frequency of the transmitted signal spectrum.) The interaction function is then

$$g(\tau, \eta) = \delta(\eta)\left[\delta(\tau) + \frac{\alpha}{2}\exp(-i\theta)\,\delta\left(\tau - \frac{\tau_c}{2}\right) + \frac{\alpha}{2}\exp(i\theta)\,\delta\left(\tau + \frac{\tau_c}{2}\right)\right]. \tag{29}$$

Thus there is no correlation between values of the spreading function at different frequency shifts, while there is some correlation at different time delays up to a maximum separation of τ_c [see Figure 3(c)]. The amount of correlation depends on α. Substituting Eq. (29) into Eq. (27), the averaged ambiguity function is

$$A(\tau, \eta; t) = |\chi(\tau, \eta)|^2 + \frac{\alpha}{2}\exp(-i\theta)\,\chi^*\left(\tau - \frac{\tau_c}{4}, \eta\right)\chi\left(\tau + \frac{\tau_c}{4}, \eta\right) + \frac{\alpha}{2}\exp(i\theta)\,\chi^*\left(\tau + \frac{\tau_c}{4}, \eta\right)\chi\left(\tau - \frac{\tau_c}{4}, \eta\right). \tag{30}$$

Thus, in addition to the contribution usually present for uncorrelated scattering, there is an additional component which can either add or subtract, depending on the values of θ and the χ-function. In order to guarantee that the random subtraction not degrade the output of a <u>matched filter</u>, we require that the peaks of the latter two components of Eq. (30) not overlap the peak of the first term. This can be accomplished if the width of the ambiguity function is narrower than the separation of components, i.e.,

$$\frac{1}{W} < \frac{\tau_c}{4}, \text{ or } W > 4/\tau_c. \tag{31}$$

Thus there is a minimum signal bandwidth to employ to avoid the correlated-spreading effect. Another way of seeing this result is by noting that a signal bandwidth larger than $4/\tau_c$ covers two full cyclic variations of $C(f, t)$ [see Figure 3(c)] thereby averaging out the effects of the filter, regardless of the placement of the signal spectrum with respect to the peaks and valleys of the filter transmission. We must stress that this result has been derived only for the matched linear processor, which is not optimum under the present conditions.

Although the interaction function performs effectively a local averaging of the ambiguity funtion, it does not destroy the volume underneath it. That is, there is an uncertainty relation which holds true for A. It is

$$\iint d\tau\, d\eta\, A(\tau, \eta; t) = \iint df\, du \left| S_{\underline{x}}(f) \right|^2 \left| \underline{s}(u) \right|^2 C(f, t + u). \tag{32}$$

Here $S_{\underline{x}}(f)$ is the Fourier transform of $\underline{x}(t)$. While $C(f, t)$ is unity for uncorrelated scattering, and the standard uncertainty results, there is but little change in the volume for correlated scattering because $C(f, t)$ is a weak function of f and t, and only its average value is of importance. A notable exception to this rule is when the signal bandwidth W is smaller than $1/\tau_c$ (which is the width of dominant variations in the transmission function) and lies at a deep valley of the transmission function. If the filter variation with time is also constant over the signal duration, Eq. (32) becomes

$$\iint d\tau \, d\eta \, A(\tau, \eta \, ; \, t) \cong C(0, t) \int df \, | \, S_{\underline{x}}(f) \, |^{2} \int du \, | \, \underline{s}(u) \, |^{2}, \quad (33)$$

and very little output from the matched filter results. These catastrophic spectrum locations must be avoided if at all possible.

2. Empirical Results on the Characterization of Undersea Acoustic Channels

2.1 Qualitative Background

We now turn to a presentation of some empirical data which shed light on the ocean from the point of view which has just been developed. This transition from theory to experiment will be made smoother if we first take a rapid qualitative glance from this same viewpoint at some of the gross aspects of the physics of undersea acoustic propagation. Among these are the facts that the medium is bounded, stratified, turbulent and contaminated: each of these characteristics imposes a smear on our signals.

The two boundaries--the surface and the bottom--are both rough and give scattered (as well as specular) reflections, which is to say their presence causes smears along the time delay axis, τ of the spreading function $a(\tau, \eta)$. Furthermore, one of them--the surface--is generally in motion and because of the doppler effect, reflections from it are smeared along the frequency shift axis, η, of the spreading function. The bottom is often composed of several different layers of material (e.g., mud, sand, clay), giving rise to multiple reflections, which we here visualize as discrete blobs along the delay axis, τ.

The velocity of sound in the sea is a variable, controlled largely by temperature and pressure which are, in turn, functions of depth. One result is that moderately strong ducts are usually present: one commonly exists, for example, in the near-surface region where the temperature is affected by weather history; there is generally another at a depth of a few thousand feet where there is a crossover between the opposing effects of temperature and pressure. Each such duct is a potential source of another blob along the delay axis, τ. At greater depths, i.e., below a few thousand feet, the water temperature reaches an asymptotic value so the velocity of sound is controlled by pressure and increases monotonically with depth. Rays entering this region are refracted upward and can, under some circumstances, be returned toward the surface without touching the bottom. One of the necessary circumstances is that a ray lie within a relatively small angular cone: at too steep an angle it will strike the bottom and at too shallow an angle it won't get past the velocity minimum. Thus, this mode of propagation constitutes yet another duct: it happens to be an especially stable one because of the fact that it is formed in the depths of the ocean, an environment of great stability, and, through most of the oceans, rays travelling in this duct are returned to the surface at intervals of about thirty-five miles, where they are reflected back down into another loop. In addition to stability, this mode also exhibits a low loss and is therefore especially important. It is called the 'convergence zone' or 'RSR' mode of propagation. (Refracted, Surface Reflected)

The ocean is contaminated with many scattering centres which can cause smears in both time delay and, in some cases, frequency shift. One example is marine life, which exists in a variety of forms having different sizes, social behavior, and mobilities. Entrapped bubbles near the surface are another example.

Finally, the ocean exhibits fine grain temperature (and hence velocity) fluctuations in space and time due to turbulence. The spatial autocorrelation function of this microstructure has an extent in the order of tens of centimetres. Thus, for paths of appreciable length, one must expect to observe a continuum of both types of smearing due to this cause.

Some, usually all, of these effects are present in combination: thus one can encounter simultaneously several orders of bottom-surface reflections, ducting, leakage from ducts, and reflections from schools of organisms, all smeared in both time delay, τ, and frequency shift, η.

To complete this sketch, account must be taken of the fact that man doesn't (yet) live in the sea; he only travels on it. Thus, if we are interested in applications we must, for example, include in our notion of a channel the fact that the end points are moving. This immediately raises thorny questions about stationarity; to some extent, all the important propagation parameters are functions of geography and hence, in a mobile situation, of time. Moreover, very large doppler shifts can be involved; the platform velocities may exceed 1% of the speed of sound in water, (the equivalent of several million miles per hour in electromagnetic propagation). Also, the windows through which we observe the ocean, i.e., our antennas, are affected appreciably by having to be ship mounted; e.g., there are dimensional constraints on the aperture sizes we can obtain. And, except for research vessels, man's activities at sea are restricted to a very shallow region near the surface, the worst place from a propagation viewpoint.

Finally, and most relevant to the purposes at hand, there is the fact that data gathering in this environment presents some special problems. Continuous observations cannot in general be made, except at a few research facilities with fixed antenna arrays. Every measurement is highly particularized in geography, time of year, weather history, type of antenna, and type of vessel. And, lastly, the cost of ocean-going facilities is very high.

2.1 Quantitative Results

Numerical data will be given only for one way (as opposed to echo-ranging) deep water paths in the western Atlantic over distances from 10 miles to several hundred miles.

Broadly speaking, under these circumstances, and with antenna beamwidths in the range 1/10 to 1/3 radians, an impulse response duration, L, of two to three seconds will typically be observed. [Under similar conditions but with less directional antennas, some workers have observed durations as large as eight seconds.] These responses are composed of several major separate blobs, typically two or three, each of which occupies a time interval ranging from a maximum of several hundred milliseconds down to the limit of resolution afforded by the instrumentation, the typical width being of the order of tens of milliseconds. At a resolution of one millisecond, one can typically discern a total of 10 to 20 distinct arrivals in the interval L.

There are several aspects of this situation which our theoretical models do not yet embrace sufficiently well to allow us to deduce optimum operating system parameters. Specifically, the presence of a multimodal arrival structure is an embarrassment from this optimization point of view. Pending a generalization of the theory, the experimental work has concentrated on an examination of the possibility that this multiplicity of arrivals might be susceptible to adaptive recombination, a processing which has been shown* to be optimum for channels which are slowly time varying. The effectiveness to be expected of such a combiner is obviously a function of the degree of stability exhibited by the multipath structure, and an analysis** done as part of this work has shown that a suitable stability measure is the correlation coefficient between the arrival patterns excited by successive signal transmissions. Expressions were found for the performance that could be expected of an adaptive receiver for both coherent and envelope combination and for both continuous and intermittent operation. The result that is relevant to the present discussion is that an adaptive combiner offers an advantage only if the correlation coefficient between successive arrival patterns is greater than 0.99. (The separation in time between successive arrival patterns will be taken up shortly).

Experiments have been carried out in an effort to learn whether or not the sea does exhibit this degree of stability. It is evident that this measure will be a function of the resolving power of the pulse used to excite the channel: if one could use sufficiently narrow pulses, the multipath arrivals could be separated so they would not interact to cancel or reinforce one another. Then the patterns would be relatively stable in amplitude, but for such a fine grain structure, the sensitivity to travel time fluctuations would be heightened until, in the limit of an infinitely narrow pulse, it would reduce the correlation between the envelopes of successively generated patterns to zero. At the wide pulse extreme, the dual situation would be expected, i.e., the patterns would have relatively unstable amplitudes due to interactions among the arrivals before detection, but the position of the arrivals in time would appear to be relatively constant.

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* Price, R. and Green, P.E. Jr., loc. cit.

**J. A. Mullen, "Signal to Noise Ratios for Multipath Combiners," Raytheon Report to USN/USL in six parts as follows: I. Summary of Analysis (June 27, 1962); II. The Initial Transient Period of Operation (Oct. 4, 1962); III. On the Statistics of a Clipper Output (Oct. 24, 1962); IV. Analysis of the Coherent Combiner (July 20, 1962); V. Analysis of the Envelope Combiner (Oct. 2, 1962); VI. General Covariances (Dec. 4, 1962).

To date, these measurements have been carried out only at center frequencies near 5 khz, and only at pulse widths spanning the decade from one millisecond to 10 milliseconds. [A pulse repetition interval of 5 seconds was used--large enough to avoid overlap between successive arrival patterns.] The results of interest to the present discussion are that the arrival patterns became de-correlated at average rates of from 3/10 of one percent per second for 1 ms pulses to 5/10 of one percent per second for 10 ms pulses.

It has already been mentioned that the extent in delay of the multipath arrivals (i.e., L) is typically several seconds: if we wish to communicate through such a channel, we must beware of the possibility that this delay spread will cause successive signals to overlap one another at the receiver sufficiently to produce confusion. This can be prevented, in general, only by making the interval between successive signals as long or longer than the multipath spread, i.e., at least several seconds. Thus, with a 10 millisecond resolution, successive signals would be de-correlated by at least several times 3/10 of one percent, i.e., by about 1%. But this, as noted earlier, is the limiting de-correlation which can be tolerated by an adaptive combiner, and therefore its prospects for these conditions appear marginal.

However, the experimental data indicate that the rate of de-correlation will be smaller at coarser resolutions. By extrapolating these data, it appears that at resolutions corresponding to bandwidths in the low tens of hertz, the de-correlation rate will be found to be under 0.1% per second--sufficiently small to make adaptive combination a very good possibility.

For an uncorrelated-scattering channel having a unimodal scattering function of extent L in time, we know that for matched filter reception, the signal bandwidth, W, should be less than or equal to $\frac{1}{L}$ in order to maximize the average output of the receiver (at the possible expense of range resolution). This follows from the fact that for such a channel the average output from a matched filter receiver is given by the convolution of the scattering function and the signal's ambiguity function, (see eq. 22) and to maximize that output, we must choose an ambiguity function whose extent is contained within the scattering function so that total overlap occurs. The extent of the ambuiguity function in time is of the order $\frac{1}{W}$, and hence the inequality follows. (An alternative viewpoint which may be more attractive intuitively is that if we cannot combine the arrivals in a controlled manner, which the assumption of uncorrelated scattering implies, then the next best thing with linear processing is to at least let them combine randomly; that is to say, we ought to use a signal bandwidth sufficiently small to avoid resolving the arrival structure.) It was mentioned earlier that the typical time smear of each blob of arrivals is of the order tens of milliseconds, and it is interesting to note that the inverse, tens of hertz, lies in the same range as that indicated from considering the possibility of adaptive combination. That is to say, this suggests that for a multimodal scattering function, the extent of the individual blobs may be of at least as much interest as the total extents B and L.

It should be noted that at a bandwidth as narrow as tens of hertz, the number of resolvable arrivals will be small and so, also, will be the gain derived from combining them. It is estimated that after allowance is made for the effects of the remaining decorrelation rate and the inevitable compromises made in hardware realizations, the net gain will be very nearly zero in terms of reducing the required signal energy at the receiver. There are other advantages of an adaptive combiner, however, having to do with easing the problems of automating the receiver functions, and it is on such side-benefits that a justification of its inclusion must apparently rest.

We now turn to the frequency domain. It is appropriate to begin with something about selective fading since that is probably the characteristic most closely related to time domain behavior. Only one series of measurements have been made in this area as part of this work. They consisted of transmitting a seven hundred hertz band of white noise centered at 2000 hz from one ship to another and obtaining the power spectrum of the received signal. Two second segments and a resolution of 5 hz were used in the spectral analysis; the 5 hz resolution was chosen to smooth out the narrowly spaced frequency domain influences caused by the widely spaced major blobs (in time). Strong selective fading was observed in the spectra, and the nulls in the pattern had typical widths in the middle to high tens of cycles. This result can be shown to be consistent with the typical smears of tens of milliseconds of the major blobs mentioned earlier. The spectra were obtained at four second intervals and the selective fading pattern could be seen to persist for times of the order of one minute, a result which is consistent with the stability of the multipath arrival patterns already mentioned.

Moving completely over into the frequency domain, there are some measurements on the extent of frequency shift spreading, (i. e., B), that are of interest. In the five khz centre frequency region we have made sinewave transmissions and calculated the spectral spreading from the envelope of the received signal. This procedure depends for its justification on a Gaussian assumption which is now known to be of doubtful legitimacy in this application, but the economics which it offers in data reduction costs are very great. The results indicate a typical frequency spread for slowly moving platforms of about one hertz; there have been encountered extremes as high as five hertz and as low as the experiment's limit of resolution, 0. 1 hz.

Coupling these figures with those previously given for the time delay spread we see that the typical BL product at five khz centre frequency is two to three; i. e., at this centre frequency the channel is typically overspread.

Frequency smearing has its genesis in the doppler phenomenon and so it should be expected to diminish as the centre frequency is lowered--leading, perhaps, in the case at hand, to an underspread channel. In pursuing this possibility, measurements have been made over a range of centre frequencies extending into the low hundreds of hertz. These have not been based on the received envelope but rather have been full-fledged (expensive) spectral analyses of the complete received signal. In this phase of the work we have used paths with fixed end points in addition to making ship-borne tests. One of the fixed to fixed paths was only 1/2 mile in length and did not intercept the surface; it was included as a reference.

The results for all paths except the reference one are that the width of the frequency smear does indeed decrease with centre frequency, following an approximately linear relationship down to about 500 hz where it rather abrupt drops by an order of magnitude or more. On the reference path, no frequenc smear was discerned at centre frequencies up to 1 khz, the highest transmitt over it. This spectacular behavior is believed to be a manifestation of the dominant role played by the surface of the ocean in paths that have been considered here: one of the features of the statistics of wind-generated surface waves is that the spectrum vanishes very abruptly at low frequencies.* The cause is not fully understood, but is believed to be rooted in the (non-linear) processes of wave generation. The surface wavelength at which this occurs is approximately that which corresponds to a first order diffraction grating spacing for an acoustic frequency of 500 hz at the angles of incidence involvec in RSR paths: i. e., at lower acoustic frequencies the surface would appear smooth.

In any case, it appears that at these low frequencies the ocean does offer a channel which is underspread by a sufficient degree to give grounds for believing that it may be sufficiently reliable to be exploitable.

2. 3 Summary

It may be worthwhile to summarize this description of underwater paths with an eye to how well they match existing theoretical models. Before doing that, the particular nature of these paths ought to be reemphasized; only one-way deep-water paths have been treated; only intermediate distances have been used; no work has been done outside the Western Atlantic; medium to high transmitting and receiver spatial directivities have been used in all cases only slowly moving or fixed end points have been employed. These particular paths are among the best the sea has to offer, bad as they may seem by electromagnetic standards.

It has been shown that a typical undersea acoustic channel exhibits a very complex time delay spread: several seconds in total duration, and composed of several discrete arrival groups. Although it has not been mentioned

*Neumann, Gerhard and Pierson, Willard J. Jr., "Known and Unknown Properties of the Frequency Spectrum of a Wind-Generated Sea," in Ocean Wave Spectra, Prentice-Hall, Inc.; 1963. See pp. 9-21.

explicitly here, the same discrete behavior is exhibited in the detailed frequency domain structure. We do not at present know how to determine optimum parameters for systems which utilize channels with multimodal scattering functions. Are the maximum extents in each direction of importance, or should we be interested in the area actually covered, or, indeed, are there any simple measures like B and L in the unimodal case which have meaning here?

Most past models contain the assumption that the spreading function is uncorrelated for different time delays and/or frequency shifts. This is not generally true for undersea acoustic channels and needs generalization.

An awkward problem is posed by the matter of taking averages. We cannot expect to have more than one channel available for measurements and so we cannot, in practive, perform ensemble averages, but must instead made do with time averages. However, as pointed out earlier, the acoustic channel between two moving ships is not stationary, and so we are posed a dilemma.

Our eventual objective must be the erection of a theory which can prescribe measurements which will be sufficient to describe a channel and also deduce from that description a set of system parameters. Right now we can do neither of these very well for real channels.

2.4 Acknowledgements

The experimental work reported here was done with, as well as for, the U. S. Navy Underwater Sound Laboratory, and the following members of that laboratory's staff have been direct participants: W. L. Clearwaters, A. T. Gardener, F. J. Kingsbury, R. V. Lewis, P. F. Radics, H. R. Weaver.

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13. ABSTRACT An undersea acoustic link between two points in space is represented as a random time-varying linear filter. It can therefore be characterized by its impulse response, or alternately by various Fourier transforms, known as the transfer functions, spreading function, and bi-frequency function. All four of these descriptors are random functions of two variables. A generalization of the uncorrelated scattering model is proferred and it is shown how it can fit known ocean characteristics.

Results are presented of some measurements made of the time and frequency spreading imposed on audio frequency transmissions over open ocean underwater paths. Most of the results deal with frequency spreading; there are fine grain time domain measurements. (The major features of the time domain behavior are already part of the literature.) Results are given for several center frequencies ranging from the low hundreds to the low thousands of cycles per second. Paths between both fixed and mobile end points have been measured. The path lengths used have ranged from one-half to several hundreds of miles.

The results indicate that many of the simplifying assumptions usually made in discussions of time varying channels are not satisfied by these paths, and these points where the assumptions must be generalized are briefly discussed.

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